RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2016-19]

B.A./B.Sc. SECOND SEMESTER (January – June) 2017

Mid-Semester Examination, March 2017

Date : 15/03/2017

MATHEMATICS (Honours)

Time : 11 am– 1 pm

Paper : II

Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

Answer any two questions from Question nos. 1 – 3 : [2×4]

1. If a, b, c, d be all real numbers greater than 1, then prove that $(a+1)(b+1)(c+1)(d+1) < 8(abcd+1)$.

2. If a, b, x, y be positive real numbers, then prove that $\frac{ax+by}{a+b} > \frac{(a+b)xy}{ay+bx}$.

3. If n be a positive integer, prove that $\frac{1}{\sqrt{4n+1}} < \frac{3\cdot 7\cdot 11 ... (4n-1)}{5\cdot 9\cdot 13 ... (4n+1)} < \sqrt{\frac{3}{4n+3}}$.

Answer any one question from Question nos. 4 & 5 : [1×5]

4. If z_1, z_2 be two complex numbers, then prove that $|z_1+z_2| \le |z_1|+|z_2|$. Explain when equality occurs and also prove that $||z_1|-|z_2|| \le |z_1-z_2|$. [2+1+2]

5. a) Find arg z where $z = 1 + i\tan\frac{3\pi}{5}$. [3]

   b) Express –1–i in polar form. [2]

Answer any three questions from Question nos. 6 - 10 : [3×4]

6. Show that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+...$ converges for p>1 and diverges for $p \le 1$.

7. State and prove Cauchy's condensation test for convergence of a series of positive terms.

8. Prove that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...$ converges to log 2 but the re-arranged series $1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+...$ converges to $\frac{1}{2}\log 2$.

9. Prove that a Compact subset of $\mathbb{R}$ is closed and bounded.

10. Let K be a non-empty compact set in $\mathbb{R}$. Show that K has a least element.

Group – B

Answer any two questions from Question nos. 11 – 13 : [2×5]

11. $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$. Show that $A^3 - 6A - 9I_3 = O$. Hence obtain a matrix B such that BA = I_3. [3+2]

12. a) A be an idempotent matrix of order p. Prove that $(I_p + A)^n = I_p + (2^n - 1)A$ for all $n \in \mathbb{N}$. [3]

b) Prove that $\begin{vmatrix} 1+a_1 & 1 & 1 & 1 \\ 1 & 1+a_2 & 1 & 1 \\ 1 & 1 & 1+a_3 & 1 \\ 1 & 1 & 1 & 1+a_4 \end{vmatrix} = a_1a_2a_3a_4\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}\right)$. [2]

13. a) A and B are real orthogonal matrices of the same order and detA + detB = 0. Show that A + B is a singular matrix. [2]

b) Use elementary row operations to find inverse for $A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{pmatrix}$. [3]

Answer <u>any two</u> questions from <u>Question nos. 14 – 16</u> : [2×6]

14. Prove that every extreme point of the convex set of all feasible solutions of the system Ax = b, $x \geq 0$ corresponds to a basic feasible solution.

15. If for a B.F.S. x_B of a linear programming problem

Maximize $z = cx$

subject to $Ax = b$

$x \geq 0$

we have $z_j - c_j \geq 0$ for every column a_j of A then prove that x_B is an optimal solution.

16. Solve by Charnes Big M – method, the L.P.P. :

Minimize $z = 4x_1 + 2x_2$

subject to $3x_1 + x_2 \geq 27$

$x_1 + x_2 \geq 21$

$x_1 + 2x_2 \geq 30$

$x_1, x_2 \geq 0$

Answer <u>any one</u> question from <u>Question nos. 17 & 18</u> : [1×3]

17. $x_1 = 1, x_2 = 1, x_3 = 1$ and $x_4 = 0$ is a feasible solution of the equations

$x_1 + 2x_2 + 4x_3 + x_4 = 7$

$2x_1 - x_2 + 3x_3 - 2x_4 = 4$

Reduce the F.S to a B.F.S.

18. Prove that the set of all feasible solutions of a L.P.P. is a convex set.

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